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**Summer 2012**

**ECE 20200 : Linear Circuit Analysis II**
**School of ECE, Purdue University**

LECTURE 14

- Frequency Response $H(j\omega)$

Reference : Decarlo/Lin pp 707-714

# FREQUENCY RESPONSE OF SIMPLE CIRCUITS

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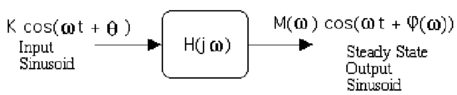
## I. WHAT IS FREQUENCY RESPONSE?

**1.** Recall Sinusoidal Steady State Analysis




(a) $M(\omega) = K \times |H(j\omega)|$ -- $\omega$-dependent gain

(b) $\varphi(\omega) = \angle H(j\omega) + \theta$ -- $\omega$-dependent phase

**2. DEFINITION:** the ***frequency response*** of a circuit/system is $H(j\omega) = H(s)\big]_{s=j\omega}$, i.e., $H(s)$ evaluated along the imaginary axis.

NOTE: for each ω, $H(j\omega)$ is a complex number:

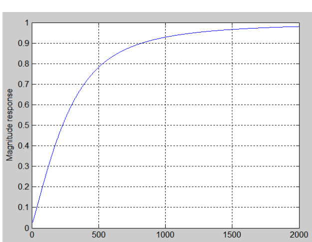
$$H(j\omega) \Leftrightarrow |H(j\omega)|\angle H(j\omega)$$

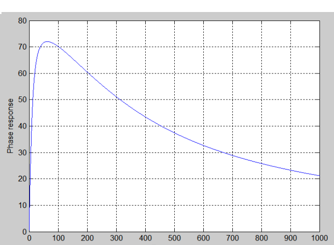
(a) $\angle H(j\omega)$ is the ***phase response***, and

(b) $|H(j\omega)|$ is the ***magnitude response*/GAIN**.

**EXAMPLE 1.** Plot the Phase and Magnitude of $H(s) = \dfrac{s+10}{s+400}$.

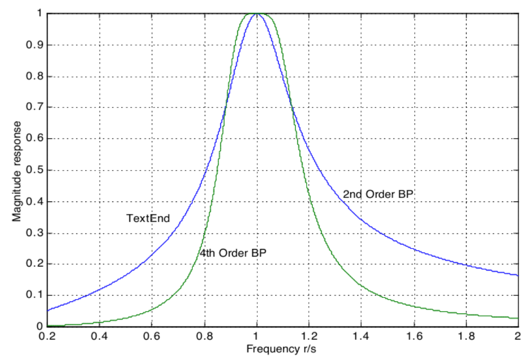
| $s = j\omega$ | $j0$ | $j\infty$ | $j10$ | $j400$ |
|---|---|---|---|---|
| $\lvert H(j\omega)\rvert$ | 0.025 | 1 | $\dfrac{10\sqrt{2}}{\sqrt{10^2+400^2}} = \dfrac{10\sqrt{2}}{400}$ | $\dfrac{400}{400\sqrt{2}} = \dfrac{1}{\sqrt{2}}$ |
| $\angle H(j\omega)$ | 0 | 0 | $\angle\dfrac{10+j10}{400+j10} = 45^o - 0 = 45^o$ | $\angle\dfrac{10+j400}{400+j400} = 90^o - 45^o = 45^o$ |




```
n = [1 10];  %  numerator coefficients
d = [1  400];  %  denom coefficients
w=0.1:0.1:2000;  % Frequency range
h = freqs(n,d,w);  %  Freq response calculation
figure(1) ;plot(w,abs(h)); grid ;xlabel('Frequency r/s') ;ylabel('Magnitude response')
figure(2) ;plot(w,angle(h)*180/pi);grid ;xlabel('Frequency r/s') ; ylabel('Phase response')
```

**EXAMPLE 2.** A ***band pass* (BP)** transfer function is one that passes frequencies in a (narrow) frequency band and eliminates/attenuates-significantly those freqs outside the band.




**(i)** 2nd order **BP** transfer function (blue curve):

$$H_1(s) = \frac{0.25s}{s^2+0.25s+1} = \frac{0.25s}{(s+0.125)^2+(0.992)^2}$$

**(ii)** 4th order **BP** transfer function (green curve):

$$H_2(s) = \frac{0.0625s^2}{s^4+0.35355s^3+2.0625s^2+0.35355s+1}$$

$$= \frac{\sqrt{0.0625}s}{(s+0.0962)^2+(1.0884)^2} \times \frac{\sqrt{0.0625}s}{(s+0.08058)^2+(0.91164)^2}$$

**Remark:** $s = j\omega = 0$ and $s = j\omega = \infty$ are the two most important frequencies:

(a) $H_1(0) = H_1(j\infty) = 0$ (magnitude is zero at dc and infinity)

(b) $H_2(0) = H_2(j\infty) = 0$

**(iii)** Plot $|H(j\omega)|$ using the following MATLAB code:

```
»n1 = 0.25*[1 0];  %  numerator coefficients H1
»d1 = [1  0.25  1];  %  denom coefficients H2
»n2 = 0.0625*[1 0 0];  %  numerator coefficients H2
»d2 = [1 3.5355e-01 2.0625 3.5355e-01 1];
»w=0.2:0.005:2;  % Frequency range
»h1 = freqs(n1,d1,w);  %  Freq response calculation
»h2 = freqs(n2,d2,w);
»plot(w,abs(h1),w,abs(h2))
»grid
»xlabel('Frequency r/s')
»ylabel('Magnitude response')
»gtext('2nd Order BP')
»gtext('4th Order BP')
```

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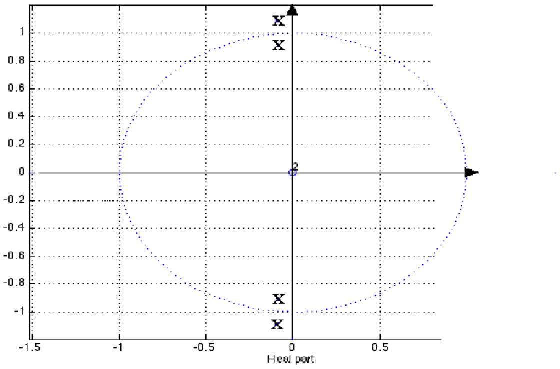
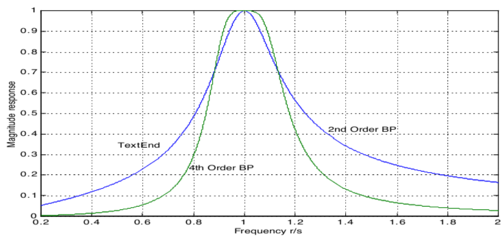
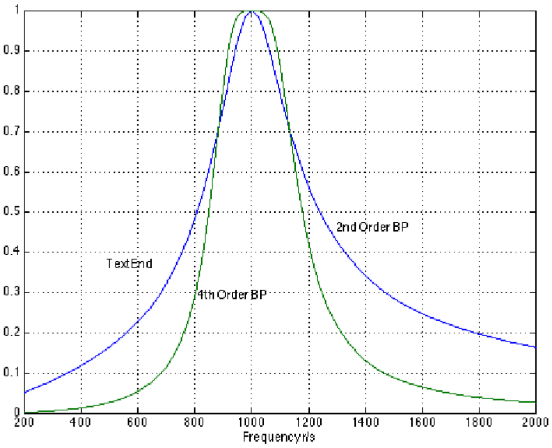
**(b) Qualitative Analysis Using Pole-zero plot**. Imagine, as you walk along the imaginary axis, what happens to your height as you near a pole, a double pole, and then as you near a zero.



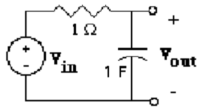

## (c) The Idea of Frequency Scaling:

```
»Kf = 1000;  %  Frequency Scale Factor
»n1 = 0.25*[1/Kf 0];  %  s → s/Kf in num coefficients of H1
»d1 = [1/Kf^2  0.25/Kf  1];  %  s → s/Kf in denom coefficients of H1
»n2 = 0.0625*[1/Kf^2 0 0];  %  s → s/Kf in num coefficients of H2
»d2 = [1/Kf^4 3.5355e-01/Kf^3 2.0625/Kf^2 3.5355e-01/Kf 1];
»w=0.2*Kf:1:2*Kf;  %  Freq range is scaled up proportionately
»h1 = freqs(n1,d1,w);
»h2 = freqs(n2,d2,w);
»plot(w,abs(h1),w,abs(h2))
»grid
»xlabel('Frequency r/s')
»ylabel('Magnitude response')
»gtext('2nd Order BP')
»gtext('4th Order BP')
```




**EXAMPLE 3.** Magnitude response $|H(j\omega)|$ of three (so called) normalized **low pass (LP) Butterworth** filter transfer functions:

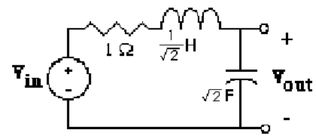
**(a) First Order Normalized LP Butterworth**




$$H(s)=\frac{1}{s+1}$$

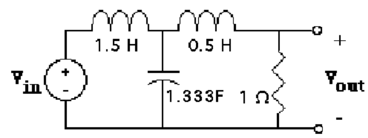
**(b) 2[nd] Order Normalized Butterworth (how many ∞ zeros)**

$$H(s)=\frac{\frac{1}{LC}}{s^2+\frac{R_s}{L}s+\frac{1}{LC}}=\frac{1}{s^2+\sqrt{2}\,s+1}$$




**(c) 3[rd] Order Normalized Butterworth (how many ∞ zeros)**

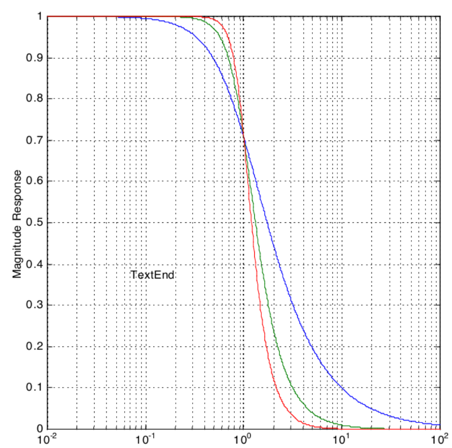
$$H(s)=\frac{1}{s^3+2s^2+2s+1}$$



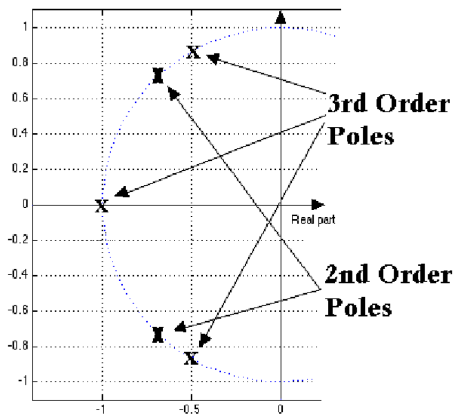

**(d)** MATLAB Code for frequency response calculation given H(s):

```
»w = logspace(-2,2,800);
»n1 = 1; d1 = [1  1];
»n2 = 1; d2 = [1  sqrt(2)  1];
»n3 = 1; d3 = [1  2  2  1];
»h1 = freqs(n1,d1,w);
»h2 = freqs(n2,d2,w);
»h3 = freqs(n3,d3,w);
»semilogx(w,abs(h1),w,abs(h2),w,abs(h3))
»grid
»xlabel('Normalized rad frequency')
»ylabel('Magnitude Response')
```

**(e) Magnitude Response Plots (Note transitions of gain from 1 to near 0)**



**(f) Pole-Zero Plot of 2[nd] and 3[rd] Order (Normalized) Butterworth Filters—Notes: How many infinite zeros? Poles lie on the unit circle in left half complex plane. Are such filters stable??**



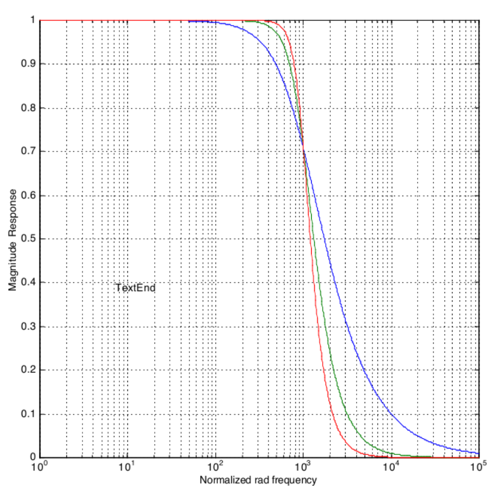

**EXAMPLE 4.** Plot of $|H(j\omega)|$ of three low pass Butterworth transfer functions of frequency scaled by $K_f = 1000$:

$$H_{new}(s) = H_{old}\left(\frac{s}{K_f}\right) = \frac{1}{\left(\frac{s}{1000}\right)+1}$$

$$H_{new}(s) = H_{old}\left(\frac{s}{K_f}\right) = \frac{1}{\left(\frac{s}{1000}\right)^2+\sqrt{2}\left(\frac{s}{1000}\right)+1}$$

$$H_{new}(s) = H_{old}\left(\frac{s}{K_f}\right) = \frac{1}{\left(\frac{s}{1000}\right)^3+2\left(\frac{s}{1000}\right)^2+2\left(\frac{s}{1000}\right)+1}$$

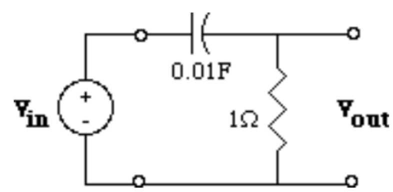
**Note: Same Plots as before with cut-off frequency at 1000.**




**EXAMPLE 5.** Frequency response of the (**High Pass-HP**) RC circuit; Capacitor has hight impedance for low frequencies and low impedance for high frequencies.

(a) Transfer function: $H(s) = \dfrac{s}{s+\dfrac{1}{C}} = \dfrac{s}{s+100}$ by V-division.

(i) One pole at $s = -100$; one zero at $s = 0$.



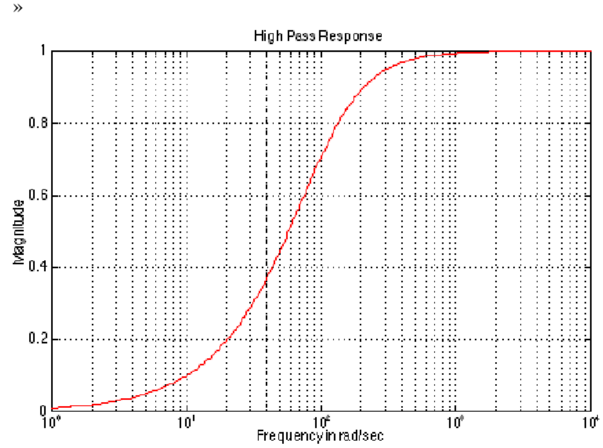

(b) $s = j\omega$, $H(j\omega) = \dfrac{R}{R+\dfrac{1}{j\omega C}} = \dfrac{j\omega CR}{j\omega CR+1} = \dfrac{j\dfrac{\omega}{100}}{j\dfrac{\omega}{100}+1}$

**STEP 2. ASYMPTOTIC BEHAVIOR: IMPORTANT FREQUENCIES ARE: $0, \infty$, AND ????**

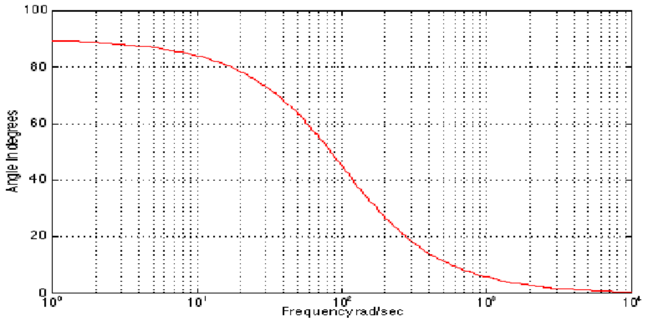
| | | |
|---|---|---|
| $\omega \to \infty$ | $\lvert H(j\omega)\rvert = \left\lvert \dfrac{R}{R+\dfrac{1}{j\omega C}} \right\rvert \to 1$ | $\angle H(j\omega) \to 0$ |
| $\omega \to 0$ | $\lvert H(j\omega)\rvert = \left\lvert \dfrac{j\omega CR}{j\omega CR+1} \right\rvert \to 0$ | $\angle H(j\omega) \to 90^o$ |
| $\omega \to 100$ | $\lvert H(j\omega)\rvert = \left\lvert \dfrac{j\dfrac{\omega}{100}}{j\dfrac{\omega}{100}+1} \right\rvert \to \dfrac{1}{\sqrt{2}}$ | $\angle H(j\omega) \to 45^o$ |

**STEP 3. PLOTS FROM MATLAB:**

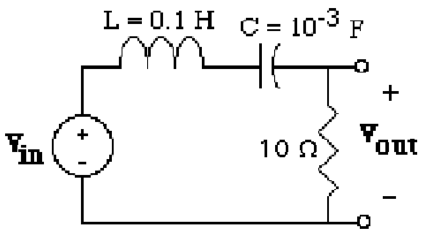
```
»w = logspace(0,4,500);
»H = freqs([1 0],[1 100],w);
»semilogx(w,abs(H))
»grid
»xlabel('Frequency in rad/sec')
»ylabel('Magnitude')
»
```



```
»semilogx(w,180*angle(H)/pi)
»grid
»xlabel('Frequency rad/sec')
»ylabel('Angle in degrees')
```




**EXAMPLE 6.** Frequency response of the (Band Pass-BP) RLC circuit




**STEP 1: CALCULATION OF TRANSFER FUNCTION:** By V-division,

$$H(s)=\frac{V_{out}(s)}{V_{in}(s)}=\frac{R}{R+Ls+\frac{1}{Cs}}=\frac{\frac{R}{L}s}{s^2+\frac{R}{L}s+\frac{1}{LC}}$$

**NOTE**: a finite zero at s = 0 and one infinite zero;

two finite poles at $s=-50\pm j86.6$

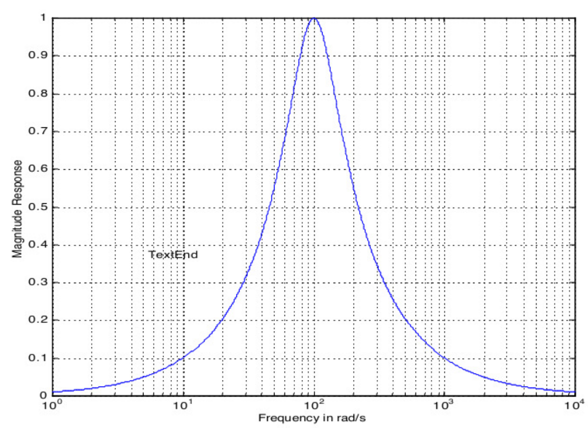
**STEP 2: IMPORTANT FREQUENCIES AND ASYMPTOTIC BEHAVIOR**

| ω (rad/sec) | $H(j\omega)=\dfrac{j\frac{\omega}{L}}{\frac{1}{LC}-\omega^2+j\frac{\omega}{L}}=\dfrac{j10\omega}{10^4-\omega^2+j10\omega}$ |
|---|---|
| $\omega\to 0$ | $\lvert H(j\omega)\rvert\angle H(j\omega)\to 0\angle 90^o$ |
| $\omega\to\infty$ | $\lvert H(j\omega)\rvert\angle H(j\omega)\to 0\angle 0^o$ |
| $\omega\to 100$ | $\lvert H(j\omega)\rvert\angle H(j\omega)\to 1\angle 0^o$ |
| ?? | $\lvert H(j\omega)\rvert\angle H(j\omega)\to\frac{1}{\sqrt{2}}\angle 45^o$ |
| ?? | $\lvert H(j\omega)\rvert\angle H(j\omega)\to\frac{1}{\sqrt{2}}\angle -45^o$ |

**Hint**: for what values of $\omega$ are the magnitudes of the real and imaginary parts of the denominator equal?

**STEP 3. PLOTS FROM MATLAB:**

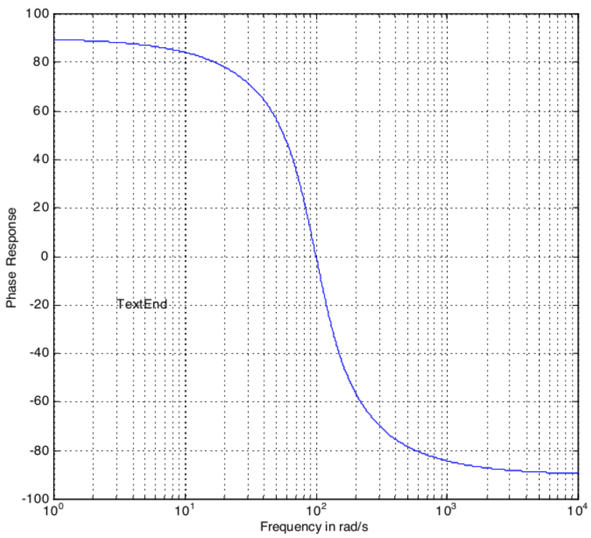
```
w = logspace(0,4,1000);
R = 10; L = 0.1; C = 1e-3;
n = [R/L 0];
d = [1 R/L  1/(L*C)];
h = freqs(n,d,w);
semilogx(w,abs(h))
grid
xlabel('Frequency in rad/s')
ylabel('Magnitude Response')
```



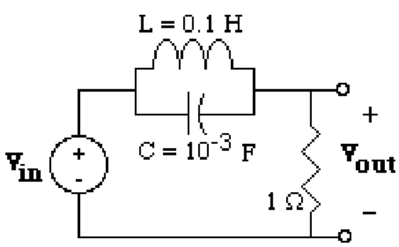

**NOTE**: a finite zero at s = 0 and one infinite zero;

two finite poles at $s=-50\pm j86.6$

```
»semilogx(w,angle(h)*180/pi)
»grid
»xlabel('Frequency in rad/s')
»ylabel('Phase Response')
```




**EXAMPLE 7.** Frequency response of the (**Band Reject-BR**) RLC circuit: Passes most frequencies with little or no attenuation and rejects a small band of frequencies.

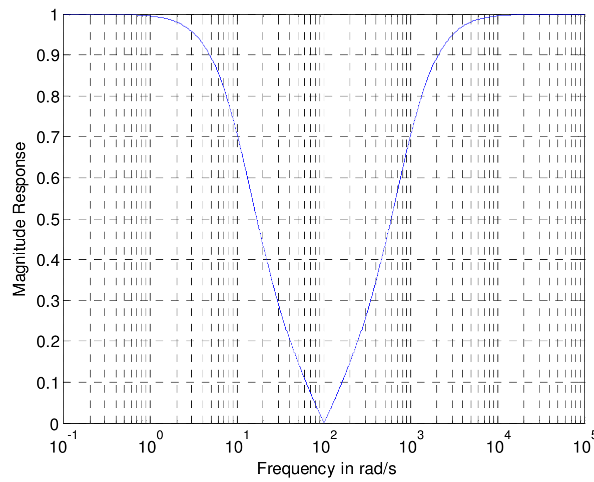


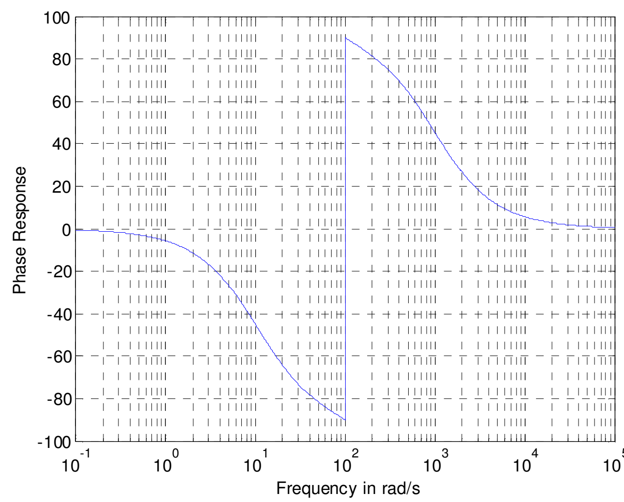

**STEP 1: CALCULATION OF TRAMSFER FUNCTION:** By V-division,

$$H(s)=\frac{R}{R+\dfrac{1}{Cs+\dfrac{1}{Ls}}}=\frac{s^2+\dfrac{1}{LC}}{s^2+\dfrac{1}{RC}s+\dfrac{1}{LC}}$$

**Remark:** Zeros at $\frac{\pm j}{\sqrt{LC}}=\pm j100$, frequencies around $\pm j100$ are mostly gone:

$$H(j\omega)=\frac{\dfrac{1}{LC}-\omega^2}{\dfrac{1}{LC}-\omega^2+\dfrac{j\omega}{RC}}$$




```
w = logspace(-1,5,1000);
R = 1; L = 0.1; C = 1e-3;
n = [1 0 1/(L*C)];
d = [1 1/(R*C)  1/(L*C)];
h = freqs(n,d,w);
figure(1)
semilogx(w,abs(h))
grid
xlabel('Frequency in rad/s')
ylabel('Magnitude Response')
figure(2)
semilogx(w,angle(h)*180/pi)
grid
xlabel('Frequency in rad/s')
ylabel('Phase Response')
```